Name ______________________________ Date ___________________

Pythagorean Theorem in Sports

Activity 1: Pythagorean Theorem in Baseball

Directions: Measure the distance between each of the bases using the yard – stick provided. Then convert your measurements to feet and record your measurements in the table below.

Distances in Feet (1 yard = 3 feet)			
Home plate to 1st base	1st base to 2nd base	2nd base to 3rd base	3rd base to home plate

Sketch a diagram of the baseball diamond in the space below. Label your diagram with all the bases using the measurements you found above.

*Once you have completed drawing and labeling your diagram of the baseball diamond, answer the following questions. You must **show your work** to receive full credit.*

1) Given the distance between each base, how far would the catcher have to throw the ball from home plate to second base? (i.e. find the distance the catcher would have to throw the ball from home plate to second base) Record your answers in feet. Simplify any radicals.

2) According to www.MLB.com, the official site of Major League Baseball, the pitcher's mound is 60 feet 6 inches from home plate. Given this information, is the pitcher's mound the midpoint of the diagonal from home plate to second base? (Use your answer to number 1 to help solve this problem).

3) Using your answer to number 2 along with the properties of the diagonals of a square, is the pitchers mound located directly in the center of the baseball diamond? How do you know?

4) What is the shortest distance, to the nearest tenth of a foot, between first base and third base?

5) Find the distance of a throw made from the catcher approximately 3 feet behind home plate in an attempt to throw out a runner trying to steal second base.

6) What is the total distance around a baseball diamond (i.e. what is the perimeter)?

7) What is the area of the baseball diamond? How does the area of a square relate to its side length? Explain your answer.

8) Is the numerical value of the area of a square always greater than the numerical value of the perimeter of a square? (i.e. For what values will the perimeter, $4s$, be greater than or equal to the area, s^2). Explain why or why not.

9) What happens to the perimeter and area of the baseball diamond if the original side lengths are doubled?

10) What happens to the perimeter and area of the baseball diamond if the sides are reduced to one – third the original length?

11) A little league baseball field has smaller dimensions than a professional baseball field. Given the distance between the catcher and second base is 50 feet, and the distance between the catcher and first base is 40 feet, what is the distance between first base and second base?

12) Number 8 is an example of a Pythagorean Triple. Identify and list at least two other Pythagorean Triples. Explain how you derived them.

ANSWER KEY:

1) Given the distance between each base, how far would the catcher have to throw the ball from home plate to second base? (i.e. find the distance the catcher would have to throw the ball from home plate to second base) Record your answers in feet. Simplify any radicals.

Using the Pythagorean Theorem we have the following:

$$90^2 + 90^2 = (distance\ to\ 2nd)^2$$
$$8{,}100 + 8{,}100 = (distance\ to\ 2nd)^2$$
$$16{,}200 = (distance\ to\ 2nd)^2$$
$$distance\ to\ 2nd = \sqrt{16,200} = \sqrt{25 * 648} = 5\sqrt{36 * 18}$$
$$distance\ to\ 2nd = 5 * 6\sqrt{18} = 30\sqrt{9 * 2} = 3 * 30\sqrt{2}$$
$$distance\ to\ 2nd = 90\sqrt{2} \approx 127.28 \text{ feet}$$

2) According to www.MLB.com, the official site of Major League Baseball, the pitcher's mound is 60 feet 6 inches from home plate. Given this information, is the pitcher's mound the midpoint of the diagonal from home plate to second base? (Use your answer to number 1 to help solve this problem).

From number 1, we know the distance from home plate to second base is exactly $90\sqrt{2}$ feet. If the pitcher's mound were the midpoint of this distance, it would be exactly half the distance from home plate to second base.

$$\left(90\sqrt{2}\right) \div 2 = 45\sqrt{2} \approx 63.64 feet$$

Since we are told that the pitcher's mound is 60 feet 6 inches from home plate, it is not the midpoint of the diagonal because if it were the midpoint, it would be $45\sqrt{2}$ feet away from home plate.

3) Using your answer to number 2 along with the properties of the diagonals of a square, is the pitchers mound the center of the baseball diamond? How do you know?

No, the pitchers mound is not the center of the baseball diamond. The center of the baseball diamond would be the midpoint of either of the diagonals of the square (i.e. the center of the baseball diamond is $45\sqrt{2}$ feet away from 1^{st} base, 2^{nd} base, 3^{rd} base, and home plate). However, the pitcher's mound is positioned at 60 feet 6 inches away from home plate. This means that the pitchers mound is approximately 3.04 feet in front of the center of the baseball diamond.

4) What is the shortest distance, to the nearest tenth of a foot, between first base and third base?

Using the Pythagorean Theorem we have the following:

$$90^2 + 90^2 = (distance\ between\ 1st\ and\ 3rd)^2$$
$$8{,}100 + 8{,}100 = (distance\ between\ 1st\ and\ 3rd)^2$$
$$16{,}200 = (distance\ between\ 1st\ and\ 3rd)^2$$
$$distance\ between\ 1st\ and\ 3rd = \sqrt{16{,}200} = \sqrt{25 * 648} = 5\sqrt{36 * 18}$$
$$distance\ between\ 1st\ and\ 3rd = 5 * 6\sqrt{18} = 30\sqrt{9 * 2} = 3 * 30\sqrt{2}$$
$$distance\ between\ 1st\ and\ 3rd = 90\sqrt{2} \approx 127 \text{ feet}$$

5) Find the distance of a throw made from the catcher approximately 3 feet behind home plate in an attempt to throw out a runner trying to steal second base.

In order to solve this problem, we must find the distance between second base and home plate. Then, we must add 3 feet onto this value because the catcher is standing 3 feet behind home plate.

Using the Pythagorean Theorem we have the following:

$$90^2 + 90^2 = (distance\ between\ 2nd\ and\ home)^2$$
$$8{,}100 + 8{,}100 = (distance\ between\ 2nd\ and\ home)^2$$
$$16{,}200 = (distance\ between\ 2nd\ and\ home)^2$$
$$distance\ between\ 2nd\ and\ home = \sqrt{16{,}200} = \sqrt{25 * 648} = 5\sqrt{36 * 18}$$
$$distance\ between\ 2nd\ and\ home = 5 * 6\sqrt{18} = 30\sqrt{9 * 2} = 3 * 30\sqrt{2}$$
$$distance\ between\ 2nd\ and\ home = 90\sqrt{2} \approx 127 \text{ feet}$$
$$Distance\ of\ a\ throw\ made\ from\ a\ catcher \approx 127 + 3 = 130 \text{ feet}$$

6) What is the total distance around a baseball diamond (i.e. what is the perimeter)?

$$Perimeter = 4s, where\ s\ is\ the\ side\ length$$
$$Perimeter = 4(90) = 360\ feet$$

7) What is the area of the baseball diamond? How does the area of a square relate to its side length? Explain your answer.

$$Area = s^2 = (side\ length)^2$$
$$Area = 90^2 = 8{,}100\ square\ feet$$

The area of a square is the side length squared. Therefore, to find the side length given the area, you must take the square root of the area.

8) Is the numerical value of the area of a square always greater than the numerical value of the perimeter of a square? (i.e. For what values will the perimeter, 4*s* be greater than or equal to the area, s^2). Explain your answer.

No the numerical value of the area of a square will not always be greater than the numerical value of the perimeter of a square. Take for example a square with side lengths equal to 1. The numerical value of the area of the square would be equal to 1, and the numerical value of the perimeter of the square would be equal to 4.

$$s^2 - 4s = 0$$
$$s(s-4) = 0$$
$$s = 0, and\ s = 4$$

So, for the side lengths between s = 0 and s = 4 but not including s = 0 (because area and perimeter cannot be zero), the perimeter will be greater than or equal to the area.

$$s = 1: p = 4(1) = 4, a = 1^2 = 1$$
$$s = \frac{1}{2}: p = 4\left(\frac{1}{2}\right) = 2, a = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$
$$s = 2: p = 4(2) = 8, a = 2^2 = 4$$
$$s = 3: p = 4(3) = 12, a = 3^2 = 9$$
$$s = 4: p = 4(4) = 16, a = 4^2 = 16$$

9) What happens to the perimeter and area of the baseball diamond if the original side lengths are doubled?

If the side lengths are doubled, we have a baseball diamond with side lengths equal to 180.

$$p = 4(180) = 720\ feet$$
$$a = (180)^2 = 32{,}400\ square\ feet$$

Notice, by doubling the original side lengths, the perimeter has doubled as well, but the area has quadrupled.

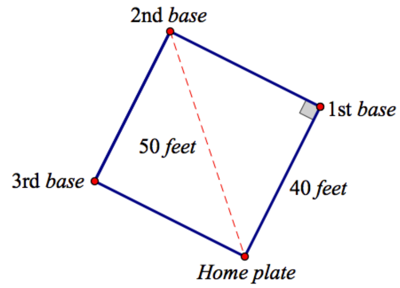
10) What happens to the perimeter and area of the baseball diamond if the sides are reduced to one-third the original length?

If the side lengths are reduced by one – third the original length, we will have a baseball diamond with side lengths equal to 30.

$$p = 4(30) = 120\ feet$$
$$a = (30)^2 = 900\ square\ feet$$

Notice, by reducing the side lengths to one – third the original side lengths, the perimeter has reduced by one – third as well, but the area has reduced by one – ninth.

11) A little league baseball field has smaller dimensions than a professional baseball field. Given the distance between the catcher and second base is 50 feet, and the distance between the catcher and first base is 40 feet, what is the distance between first base and second base?



$$40^2 + \ (distance\ from\ 1st\ to\ 2nd)^2 = 50^2$$
$$1{,}600 + (distance\ from\ 1st\ to\ 2nd)^2 = 2{,}500$$
$$(distance\ from\ 1st\ to\ 2nd)^2 = 900$$
$$distance\ from\ 1st\ to\ 2nd = \sqrt{900} = 30\ feet$$

12) Number 8 is an example of a Pythagorean Triple. Identify and list at least two other Pythagorean Triples. Explain how you derived them.

If given a Pythagorean Triple, then multiplying that triple by a constant produces another Pythagorean Triple. The Pythagorean Triple in number 8 is a multiple of 10 of the Pythagorean Triple (3, 4, 5). Another Pythagorean Triple may be (6, 8, 10) and (15, 20, 25).